Chapter 5: Diffraction and Beam Formation Using Arrays
B-mode Imaging

\[ p\left( t - \frac{2z'}{c} \right) = A\left( t - \frac{2z'}{c} \right) \cos(2\pi f_0 \left( t - \frac{2z'}{c} \right)) \]
Linear Scanning
Beam Formation Using Arrays

Focusing:
Curved Linear Scanning
Sector Steering
Focusing $\leftrightarrow$ Beam Formation

- To form a beam of sound wave such that only the objects along the beam direction are illuminated and possibly detected.
**Nomenclature**

- **x:** Lateral, azimuthal, scan
- **y:** Elevational, non-scan
- **z:** Axial, range, depth

- Beam pattern
- Radiation pattern
- Diffraction pattern
- Focusing pattern
Beamforming

- Manipulation of transmit and receive apertures.
- Trade-off between performance/cost to achieve:
  - Steer and focus the transmit beam.
  - Dynamically steer and focus the receive beam.
  - Provide accurate delay and apodization.
  - Provide dynamic receive control.
Imaging Model

transmitter \rightarrow\text{transducer} \rightarrow\text{propagation in the body} \rightarrow\text{transducer} \rightarrow\text{receiver} \rightarrow\text{display}

A-scan:

\[ V(t) = k \iiint R(x', y', z') e^{-2\beta z'} B(x', y', z') p(t - \frac{2z'}{c}) \, dx' \, dy' \, dz' \]

B-scan:

\[ S(x, t) = k \iiint R(x', y', z') B(x' - x, y', z') p(t - \frac{2z'}{c}) \, dx' \, dy' \, dz' \]

Scanning \rightarrow\text{Convolution}

(Correlation vs. Convolution)
Imaging Model

\[ p(t - \frac{2z'}{c}) = A(t - \frac{2z'}{c}) \cos(2\pi f_0 (t - \frac{2z'}{c})) \]

Ideally,

\[ S(x, t) = R(x, y_0, c t / 2) \]

In practice,

\[ B(\lambda) : \text{determined by diffraction} \]

\[ A(\lambda) : \text{determined by transducer bandwidth} \]
Beam Formation as Spatial Filtering

- Propagation can be viewed as a process of linear filtering (convolution).
- Beam formation can be viewed as an inverse filter (or others, such as a matched filter).
Imaging Model

\[
S(x, y, z) = H(x, y, z) \bullet \bullet \bullet R(x, y, z) \mid_{z = \alpha/2}
\]
Diffraction from 1D Apertures

- Free space Green’s function:

\[ p(R) = A_0 \frac{e^{ikR}}{R} \]

Continuous wave (CW, single frequency)

\[ p(x', z) = \int_{-a}^{a} \frac{e^{jkd(x, x')}}{d(x, x')} dx \]

- \( d(x, x') \) is the distance between the points \( x \) and \( x' \) in the aperture plane and the observation plane, respectively.

\( z = 0 \) is the aperture plane.

\( x = 0 \) and \( a \) are points on the aperture boundary.
In the Fresnel Region

\[ z^2 \gg (x - x')^2 \]

\[ d(x, x') = z \left(1 + \frac{(x - x')^2}{z^2}\right)^{1/2} \approx z + \frac{(x - x')^2}{2z} \]

\[ p(x', z) \approx \frac{1}{z} \int_{-a}^{a} e^{jkz} e^{j(k(x - x'))^2/2z} dx = \frac{e^{jkz}}{z} \int_{-a}^{a} e^{-jkxx'/z} e^{jkx^2/2z} dx \]

\[ C(x) = |C(x)| e^{j\theta(x)} \]

\[ p(x', z) \approx \frac{e^{jkz}}{z} \int_{-a}^{a} C(x) e^{-jkxx'/z} e^{jkx^2/2z} dx \]
Focusing in the Far Field (or Focal Point)

\[ \frac{ka^2}{2z} \ll 1 \]

\[ p(x', z) \approx \frac{e^{jkz} e^{jka^2/2z}}{z} \int_{-a}^{a} C(x) e^{-jka^2/2z} dx = \frac{e^{jkz} e^{jka^2/2z}}{z} F.T.[C(x)] \]

Aperture \( \leftrightarrow \) (F.T.) \( \rightarrow \) Radiation Pattern

When not in the far field \( \rightarrow \) effective aperture function

\[ C(x) = |C(x)| e^{-jka^2/2z} \]
Focusing: An Acoustic Lens

\[ C(x) = |C(x)|e^{-jkx^2/2z} \]

When out of the fixed focal point:

\[ C'(x) = |C(x)|e^{\frac{jkx^2}{2}\left(\frac{1}{z} - \frac{1}{z_0}\right)} \]
Radiation Pattern of a Rectangular Aperture

\[ |p(x', z)| = \int_{-a}^{a} e^{-j k x' x / z} \, dx = \frac{1}{j k x' / z} \left[ e^{j k x' a / z} - e^{-j k x' a / z} \right] = 2a \sin \left( \frac{k x' a}{z} \right) \]
Lateral Resolution

- Frequency $\uparrow$
- Aperture size $\uparrow$
- -3 dB, -6 dB, -10 dB, -20 dB,…etc.
CW to Wideband

\[ B(x', z) = \int T(x', z, \omega) R(x', z, \omega) A(\omega) d\omega \]

\[ A(t) \leftrightarrow A(\omega) \]

\[ p(t - \frac{2z'}{c}) = A(t - \frac{2z'}{c}) \cos(2\pi f_0 (t - \frac{2z'}{c})) \]
Radiation Pattern
Unfocused

Focused

Focused at $\frac{1}{2}$ range

Focused with twice the aperture
Axial Intensity

- Unfocused
- Focused
- Focused at ½ range
- Focused with 2X aperture
Diffraction and Propagation Delays
CW to Pulse Wave

\[ p(x', z, t) = \int_{-a}^{a} A(t - \tau(x, x', z)) \cos(\omega_0(t - \tau(x, x', z))) \, dx \]

\[ \tau(x, x', z) = \left( (x - x')^2 + z^2 \right)^{\frac{1}{2}} / c \]

In Fresnel region

\[ \tau(x, x', z) \approx \frac{z}{c} + \frac{(x - x')^2}{2zc} \]

Apply focusing delays

\[ \tau'(x, x', z) = \frac{x^2}{2zc} \]

\[ p(x', z, t) = \int_{-a}^{a} A(t - \tau(x, x', z) + \tau'(x, x', z)) \cos(\omega_0(t - \tau(x, x', z) + \tau'(x, x', z))) \, dx \]

\[ = \int_{-a}^{a} A\left(t - \frac{z}{c} + \frac{xx'}{zc} - \frac{x'^2}{2zc}\right) \cos(\omega_0\left(t - \frac{z}{c} + \frac{xx'}{zc} - \frac{x'^2}{2zc}\right)) \, dx \]
CW to Pulse Wave

\[ x' = 0 \]
\[ p(0, z, t) = \int_{-a}^{a} A(t - \frac{Z}{c}) \cos(\omega_0(t - \frac{Z}{c})) \, dx = 2aA(t - \frac{Z}{c}) \cos(\omega_0(t - \frac{Z}{c})) \]

\[ x' \neq 0 \]
\[ p(x', z, t) = \text{Re} \left( \int_{-a}^{a} A(t - \frac{Z}{c} + \frac{xx'}{zc} - \frac{x'^2}{2zc}) \, dx \right) \, e^{j\omega_0 \left( \frac{Z}{c} + \frac{xx'}{zc} - \frac{x'^2}{2zc} \right)} \]
\[ = \text{Re} \left( e^{j\omega_0 \left( \frac{Z}{c} - \frac{x'^2}{2zc} \right)} \int_{-a}^{a} A(t - \frac{Z}{c} + \frac{xx'}{zc} - \frac{x'^2}{2zc}) \, dx \right) \, e^{j\omega_0 \frac{xx'}{zc}} \]

\[ \frac{xx'}{zc} - \frac{x'^2}{2zc} \approx 0 \]
\[ p(x', z, t) = \text{Re} \left( e^{j\omega_0 \left( \frac{Z}{c} - \frac{x'^2}{2zc} \right)} A(t - \frac{Z}{c}) \int_{-a}^{a} e^{j\omega_0 \frac{xx'}{zc}} \, dx \right) \]
Beam Formation Using Arrays

\[ O(t) = \sum_{i=1}^{N} S_i \left( t - \tau \left( x_i, R, \theta \right) \right) \]
Propagating Delays

In Fresnel region

$$\tau(x_i, R, \theta) = \frac{\left( (x_i - R \sin \theta)^2 + R^2 \cos^2 \theta \right)^{1/2}}{c} = \frac{R}{c} \left( 1 + \frac{x_i^2}{R^2} - \frac{2x_i}{R} \sin \theta \right)^{1/2}$$

$$\tau(x_i, R, \theta) \approx \frac{R}{c} \left( 1 + \frac{x_i^2}{2R^2} - \frac{x_i}{R} \sin \theta - \frac{x_i^2}{2R^2} \sin^2 \theta \right)$$

$$= \frac{R}{c} \left( 1 - \frac{x_i}{R} \sin \theta + \frac{x_i^2}{2R^2} \cos^2 \theta \right) = \frac{R}{c} - \frac{x_i \sin \theta}{c} + \frac{x_i^2 \cos^2 \theta}{2Rc}$$

Effective aperture size: $2a \rightarrow 2a \cos \theta$
Propagating Delays

Transmit:

$$\tau^T (x_i, R, \theta) = -\frac{x_i \sin \theta}{c} + \frac{x_i^2 \cos^2 \theta}{2Rc}$$

Receive:

$$\tau^R (x_i, R, \theta) = \frac{2R}{c} - \frac{x_i \sin \theta}{c} + \frac{x_i^2 \cos^2 \theta}{2Rc}$$
Beam Forming Using Arrays

\[ \frac{x'}{z} \rightarrow \sin \theta \]

\[ p(\theta) = \int_{-a}^{a} C(x) e^{-j k x \sin \theta} \, dx \]

CW, Plane wave
Radiation Pattern of a Sampled Aperture (I)

\( f = \frac{\sin \theta}{\lambda} \)

\[
\sum_{i=-\infty}^{\infty} \delta(x - id) \leftrightarrow \frac{1}{d} \sum_{i=-\infty}^{\infty} \delta(f - i / d)
\]
Radiation Pattern of a Sampled Aperture (II)

\[ \sin \theta \cdot \frac{2aw}{d} \]
Interference

\[ \delta = d \sin \theta \]
Interference
Array Steering and Grating Lobes

\[
\sin \theta - \sin \theta_0 - \frac{\lambda}{d} - \sin \theta_0 + \frac{\lambda}{w}
\]

primary beam

secondary beam

2 \leq \frac{\lambda}{d}

d \leq \frac{\lambda}{2}
Grating Lobes

No Grating Lobes

With Grating Lobes
Grating Lobes (II)

\[ 2\pi f_0 \left( \tau \left( x_i, R, \theta \right) - \tau \left( x_{i+1}, R, \theta \right) \right) > \pi \]

\[
2\pi f_0 \left( \frac{R}{c} - \frac{x_i \sin \theta}{c} - \frac{R}{c} + \frac{x_{i+1} \sin \theta}{c} \right) = \frac{2\pi f_0 \sin \theta}{c} (x_{i+1} - x_i) = \frac{2\pi \sin \theta d}{\lambda} > \pi
\]

\[
\sin \theta > \frac{\lambda}{2d} \quad \Rightarrow \quad d \leq \frac{\lambda}{2}
\]
Beam Sampling

spatial frequency (aperture function)

\[
\frac{1}{\Delta \sin \theta / \lambda} \equiv \frac{1}{b} \geq 2a
\]

\[
\Delta \sin \theta \leq \frac{\lambda}{2a}
\]

radiation pattern

F.T.

sampling

F.T.

\[
\sin \theta / \lambda \rightarrow \frac{1}{2a} \rightarrow \sin \theta / \lambda
\]
Beam Sampling

One Way

Two Way

\[ \Delta \sin \theta \leq \frac{\lambda}{2a} \quad \text{and} \quad \Delta \sin \theta \leq \frac{\lambda}{4a} \]
Beam Sampling (II)

One Way:

\[ 2\pi f_0 \left( \tau (a, R, \theta_{i+1}) - \tau (-a, R, \theta_{i+1}) \right) - (\tau (a, R, \theta_i) - \tau (-a, R, \theta_i)) \]

\[ = \frac{2\pi f_0}{c} \left( 2a \Delta \sin \theta \right) = \frac{2\pi f_0}{c} \frac{2a\lambda}{2a} = 2\pi \]

\[ \Delta \sin \theta \leq \frac{\lambda}{2a} \]
2D Apertures
Diffraction for 2D Apertures

\[ p(x', y', z) = \iiint_{\text{area}} \frac{e^{i k d((x, y), (x', y'))}}{d((x, y), (x', y'))} \, dx \, dy \]

(x, y, 0) -- (x', y', z)
Diffraction for 2D Apertures

Separable aperture function:

\[ C(x, y) = C(x)C(y) \]

\[ B(x', y', z) = B(x', z) \times B(y', z) = F.T.\left[C(x)\right] \times F.T.\left[C(y)\right] \]

Circular aperture \(\rightarrow\) Jinc radiation pattern.
Two-Dimensional Transducer Arrays
Motivations: Elevational Focusing

- 1D arrays have only fixed elevational focusing capabilities. The elevational focusing quality is determined by a mechanical lens.
- 2D arrays must be used in order to have electronic, dynamic 3D focusing.
Motivations: Elevation Focusing

- Although not apparent, elevational focusing quality determines the "slice thickness" and is critical in image quality (including contrast resolution, noise characteristics and elevational resolution).
Motivations: Real-Time 3D Imaging

- Current 3D imaging (including flow imaging) is done by reconstruction using a set of 2D images. This is not real-time.
- Fully sampled (i.e., spatial Nyquist criterion is satisfied in both directions) arrays must be used to allow full 3D focusing and steering capabilities.
Complications

• For a fully sampled aperture with a typical size, the total channel count may exceed 10,000.

• As the element size decreases, electrical impedance significantly increases, thus resulting in poor signal to noise ratio.
Complications

- Inter-connection from each channel to the front-end electronics becomes complicated with 2D arrays.
Reduction of Channel Count

• Sparse array:
  – A random selection of elements are removed from the periodic dense array. Only a fraction of the original elements remains.
  – Mainlobe is un-affected. Grating lobes are avoided. Sidelobes are higher.
  – Electrical impedance is un-changed.
  – Difficult to manufacture.
Reduction of Channel Count

• 1.5D array:
  – Aperture in elevation is under-sampled.
  – Electrical impedance is reduced.
  – Total channel count is reduced.
  – Lack of elevational steering.
  – Full 3D focusing quality on the 2D image plane.
  – Also known as an-isotropic arrays.
Sampling of 1.5D Arrays

- Uniform sampling.
- Fresnel sampling.
- Geometric delays are symmetric.
Sparse Periodic Arrays

• Despite of the periodicity, grating lobes are avoided by placing them at different locations.
Sparse Periodic Arrays

- Two-way beam pattern is determined by two-way aperture function, i.e., the convolution of the transmit aperture with the receive aperture.
- By carefully choosing the aperture functions, desired response can be synthesized.
Sparse Periodic Arrays

- Only valid for CW and in-focus. 4-to-1 reduction is reasonable.
- Signal-to-noise ratio is affected.
- In general, this is a synthetic aperture approach.
- This approach can be extended to 2D.
Lowering Impedance

- Electrical impedance significantly increases (high resistance and low capacitance) with small transducer area.
- For typical system characteristics, large impedance means poor sensitivity.
- Impedance can be reduced by many methods, including using multiple layer piezoelectric material.
Multi-Layer Ceramic

- Acoustically in series and electrically in parallel.

single layer

multiple layer
Multi-Layer Ceramic

- On transmit, the acoustic output pressure is increased by a factor of $N$ (number of layers) assuming the same drive voltage.
- The output pressure is also increased by matching the impedance.
Multi-Layer Ceramic

- On receive, the received voltage is reduced by a factor of $N$ assuming the same returning pressure.
- Capacitance is increased by a factor of $N^2$, due to the parallel connection and smaller distance between plates.
- The receive improvement depends on specific situations.
Multi-Layer Ceramic

• Both KLM and finite element models have been used to predict the performance and compared to measurement results.
• An alternative method is to have multiple layers on transmit and signal layer on receive.
• The coaxial cable capacitance can be reduced by integrating front-end with transducers.
Complications for Real-Time 3D Imaging

• Channel count.
• System complexity associated with increased channel count.
• The number of beams in a 3D imaging is dramatically increased, thus reducing "frame rate".
• Parallel beam formation is desired.
Homework #2

- Computer Homework #2: Beam Formation
- Due 12:00pm 4/24/2012 by emailing to
  paichi@ntu.edu.tw;
r99945010@ntu.edu.tw
Homework #2

• Load hw2_dat.mat. In this data file, apertureU defines a uniform aperture and apertureH defines an aperture with non-uniform weighting. The spacing between two points in both cases is defined by $d_0$ in mm. The vector pulseF defines a pulse spectrum of a particular excitation with the frequency axis specified by $f_{axis}$ (in MHz). Finally, the sound propagation velocity is defined by $\text{soundV}$ in mm/usec. In all figures, please label all axes.
1. Assuming a continuous wave at 5MHz from the far field and zero incidence angle, plot the magnitude of the one-way diffraction patterns for both apertureU and apertureH (in dB). The horizontal axis should be $\sin \theta$ from $-1$ to $1$. (20%)

2. Assuming a pulse wave which has the frequency response specified by pulseF and faxis, plot the magnitude of the one-way, far-field diffraction patterns (zero incidence angle) for both apertureU and apertureH (in dB). The horizontal axis should be $\sin \theta$ from $-1$ to $1$. (20%)
3. Calculate the –6dB and –20dB mainlobe widths of the diffraction patterns obtained from 1 and 2. Comment on your answers and the sidelobe levels between two different apodization schemes. (20%)

4. Repeat 3 if the incidence angle is at 45 degrees. Justify your answers (20%)

5. Repeat 2 and 3 if the aperture is focused at 60 mm but the diffraction patterns are drawn at 55 mm and 65 mm, respectively. Comments on your answers. (20%)

6. (Bonus) Use the simulation programs to investigate dual beam formation (transmit/receive aperture, beam spacing).