Chapter 6: Real-Time Image Formation
Generic Ultrasonic Imaging System

• Transmitter:
  – Arbitrary waveform.
  – Programmable transmit voltage.
  – Arbitrary firing sequence.
  – Programmable apodization, delay control and frequency control.
Transmit Waveform

• Characteristics of transmit waveforms.
Generic Ultrasonic Imaging System

• Receiver:
  – Programmable apodization, delay control and frequency control.
  – Arbitrary receive direction.

• Image processing:
  – Pre-detection filtering.
  – Post-detection filtering.

• Full gain correction: TGC, analog and digital.

• Scan converter: various scan format.
Generic Receiver

- A/D
- Beam former
- Filtering (pre-detection)
- Envelope detection
- Filtering (post-detection)
- Adaptive controls
- Display
- Scan conversion
- Mapping and other processing
Pre-detection Filtering
Pre-detection Filtering

- Pulse shaping. (Z)
- Temporal filtering. (t)
- Beam shaping. (X’)
  - Selection of frequency range. (Z→X’)
    \[ B(x', z) = \int T(x', z, \omega) R(x', z, \omega) A(\omega) \, d\omega \]
  - Correction of focusing errors. (X→X’)

\[
|C(x)| \quad \text{F.T.} \quad |p(x', z)|
\]

\[
\text{1: } \frac{\lambda}{2a} \quad -a \quad a \quad x'/z \quad 2a
\]
Pulse-echo effective apertures

- The pulse-echo beam pattern is the multiplication of the transmit beam and the receive beam.
- The pulse-echo effective aperture is the convolution of transmit and receive apertures.

For C.W.,

\[ C(x) = |C(x)| e^{\frac{j k x^2}{2} \left( \frac{1}{R} - \frac{1}{R_0} \right)} \]
Post-Detection Filtering

- Data re-sampling (Acoustic → Display).
- Speckle reduction (incoherent averaging).
- Feature enhancement.
- Aesthetics.
- Post-processing:
  - Re-mapping (gray scale and color).
  - Digital gain.
Envelope Detection

- Demodulation based:

\[ S(t) = A(t) \cos 2\pi f_0 t = \text{Re} \left\{ A(t) e^{j2\pi f_0 t} \right\} \]

\[ A(t) = \text{LPF} \left\{ S(t) \cos 2\pi f_0 \right\} \]

\[ D(t) = \text{abs}(A(t)) \]
Envelope Detection

- Hilbert Transform

\[ S(t) + j \times H . T . \{ S(t) \} = 2A(t) e^{j2\pi f_0 t} \]

\[ D(t) = \frac{\text{abs}(S(t) + j \times H . T . \{ S(t) \})}{2} \]

\[ HT(f) = -j \text{ sgn}(f) \]

\[ ht(t) = -\frac{1}{\pi t} \]
Beam Former Design
Implementation of Beam Formation

- Delay is simply based on geometry.
- Weighting (a.k.a. apodization) strongly depends on the specific approach.
Beam Formation - Delay

• Delay is based on geometry. For simplicity, a constant sound velocity and straight line propagation are assumed. Multiple reflection is also ignored.

• In diagnostic ultrasound, we are almost always in the near field. Therefore, range focusing is necessary.
Beam Formation - Delay

• Near field / far field crossover occurs when $f_\# = \text{aperture size}/\text{wavelength}$.

• The crossover also corresponds to the point where the phase error across the aperture becomes significant (destructive).

$$\frac{a^2}{2R} = \frac{\lambda}{8}$$
Phased Array Imaging

\[ t_{rx}(x_i, R, \theta) = -\frac{x_i \sin \theta}{c} + \frac{x_i^2 \cos^2 \theta}{2Rc} \]
Dynamic Focusing

- Dynamic-focusing obtains better image quality but implementation is more complicated.
Focusing Architecture

1  →  delay line

N  →  delay line

delay controller

summation

transducer array
Delay Pattern

- Delays are quantized by sampling-period $t_s$.

\[ k_n = \text{round} \left( -\frac{x_i \sin \theta}{c t_s} + \frac{x_i^2 \cos^2 \theta}{2 R c t_s} \right) = n \Delta \tau \]
Missing Samples

Human Body

Beamformer

Delay

Delay-Change

Time

t_2

t_1
Beam Formation

\[ \Delta \tau \] 

\[ \Delta \tau \] 

\[ \Delta \tau \] 

input 

delay controller 

output

\[ n(t) \approx -\frac{x_i \sin \theta}{c \Delta \tau} + \frac{x_i^2 \cos^2 \theta}{c^2 t \Delta \tau} \]

\[ n(t_1) - n(t_2) = 1 = \frac{x_i^2 \cos^2 \theta}{c^2 \Delta \tau} \left( \frac{1}{t_1} - \frac{1}{t_2} \right) \]
Beam Formation - Delay

- The sampling frequency for fine focusing quality needs to be over $32f_0 (\gg$ Nyquist).
- Interpolation is essential in a digital system and can be done in RF, IF or BB.

\[
\Delta \tau = \frac{\Delta \theta}{2\pi f_0} \leq \frac{1}{32f_0}
\]

\[
2\pi / 32 \approx 11.25^\circ
\]
Delay Quantization

- The delay quantization error can be viewed as the phase error of the phasors.

\[ A = \sum_{n=0}^{N-1} \cos(\phi_n) \]

\[ \sigma_A^2 = \sum_{n=0}^{N-1} \left( \frac{dA}{d\phi} \right)^2 \sigma_{\phi_n}^2 \]
Delay Quantization

\[ \langle \sin^2 \phi \rangle = \frac{1}{2} \]

\[ \sigma_{\phi_n}^2 = \sigma_{\phi}^2 = \frac{\Delta \phi^2}{12} \]

\[ \sigma_A^2 = \frac{N \times \Delta \phi^2}{24} < 1 \Rightarrow \Delta \phi < \sqrt{\frac{24}{N}} \]

- N=128, 16 quantization steps per cycles are required.
- In general, 32 and 64 times the center frequency is used.
Beam Formation - Delay

- RF beamformer requires either a clock well over 100MHz, or a large number of real-time computations.
- BB beamformer processes data at a low clock frequency at the price of complex signal processing.
Beam Formation - RF

- Interpolation by 2:
Beam Formation - RF

- General filtering architecture (interpolation by m):

```
Delay
Filter 1
Filter 2
...  
Filter m-1
```

```
MUX
FIFO
```

Coarse delay control

Fine delay control
Autonomous Delay Control

Autonomous vs. Centralized

\[ A = n_0 + 1 - \phi \]
\[ \Delta n = 1 \]
\[ j = 1 \]

\[ A = A + j - \phi \]
\[ \Delta n = \Delta n + 1 \]

\[ A <= 0? \]

\[ N \]

\[ A = A + \Delta n + n_0 \]
\[ j = j + 1 \]

bump
Beam Formation - BB

\[ A(t-\tau)\cos2\pi f_0(t-\tau) \]

\[ A(t-\tau)\cos2\pi f_0(t-\tau)e^{-j2\pi fdt} \]

LPF\( (A(t-\tau)\cos2\pi f_0(t-\tau)e^{-j2\pi fdt}) \)
Beam Formation - BB

\[ I = \text{LPF} \left\{ A(t - \tau) \cos 2\pi f_0 (t - \tau) \cos 2\pi f_d t \right\} \]

\[ = \text{LPF} \left\{ \frac{A(t - \tau)}{2} \left( \cos 2\pi ((f_0 - f_d)(t - \tau) - f_d \tau) + \cos 2\pi ((f_0 + f_d)(t - \tau) + f_d \tau) \right) \right\} \]

\[ = \frac{A(t - \tau)}{2} \cos 2\pi ((f_0 - f_d)(t - \tau) - f_d \tau) \]

\[ Q = \text{LPF} \left\{ - A(t - \tau) \cos 2\pi f_0 (t - \tau) \sin 2\pi f_d t \right\} \]

\[ = \text{LPF} \left\{ \frac{A(t - \tau)}{2} \left( \sin 2\pi ((f_0 - f_d)(t - \tau) - f_d \tau) - \sin 2\pi ((f_0 + f_d)(t - \tau) + f_d \tau) \right) \right\} \]

\[ = \frac{A(t - \tau)}{2} \sin 2\pi ((f_0 - f_d)(t - \tau) - f_d \tau) \]
Beam Formation - BB

\[ BB(t) = \frac{A(t - \tau)}{2} e^{j2\pi f (t - \tau)} e^{-j2\pi f_d \tau} \]

\[ O(t) = \sum_{i=1}^{N} \frac{A(t - \tau_i + \tau_i')}{2} e^{j2\pi f (t - \tau_i + \tau_i')} e^{-j2\pi f_d (\tau_i - \theta_i)} \]
The coarse time delay is applied at a low clock frequency, the fine phase needs to be rotated accurately (e.g., by CORDIC).
ΔΣ-Based Beamformers
Why ΔΣ?

Current Problems

- High Delay Resolution -- 32 $f_0$ (requires interpolation)
- Multi-Bit Bus

ΔΣ Advantages

- High Sampling Rate -- No Interpolation Required
- Single-Bit Bus -- Suitable for Beamformers with Large Channel-Count
Conventional vs. $\Delta\Sigma$
Advantages of Over-Sampling

• Noise averaging.
• For every doubling of the sampling rate, it is equivalent to an additional 0.5 bit quantization.
• Less requirements for delay interpolation.
• Conventional A/D not ideal for single-bit applications.
Advantages of ΔΣ Beamformers

• Noise shaping.
• Single-bit vs. multi-bits.
• Simple delay circuitry.
• Integration with A/D and signal processing.
• For hand-held or large channel count devices.
Block-Diagram of the $\Delta \Sigma$ Modulator

- Over-Sampling
- Noise-Shaping
- Reconstruction

The SNR of a 32 $f_0$, 2nd-order, low-passed $\Delta \Sigma$ modulator is about 40dB.
Noise Shaped ΔΣ Modulator
Signal and Noise Transfer

Signal transfer function: \[ S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{H(z)}{1 + H(z)} \]

Noise transfer function: \[ N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1 + H(z)} \]

\[ H(z) = \frac{z^{-1}}{1 - z^{-1}} \quad \text{(Noninverting Forward-Euler SC integrator)} \]

\[ \Rightarrow S_{TF}(z) = \frac{H(z)}{1 + H(z)} = z^{-1} \]

\[ N_{TF}(z) = \frac{1}{1 + H(z)} = (1 - z^{-1}) \quad z = e^{j\omega T} = e^{j2\pi f/fs} \]

\[ N_{TF}(f) = 1 - e^{-j2\pi f/fs} = \sin\left(\frac{\pi f}{f_s}\right) \times (2j) \times (e^{-j\pi f/fs}) \]

\[ |N_{TF}(f)| = 2\sin\left(\frac{\pi f}{f_s}\right) \]
Noise Shaping Transfer Functions

• For first order noise shaping, 1.5 bits (9 dB) is gained when the sampling frequency is doubles.
• For second order noise shaping, 2.5 bits (15 dB) is gained when the sampling frequency is doubles.
Property of a $\Delta \Sigma$ Modulator

Waveform

Spectrum

$\begin{align*}
\text{Sample} & \quad 1 \quad 256 \quad 512 \quad 768 \quad 1024 \\
\text{x} & \quad 0.5 \quad 0 \quad -0.5 \\
\text{y} & \quad 0 \quad 0 \quad 0 \\
\text{x}* & \quad 0.5 \quad 0 \quad -0.5 \\
\text{Frequency} & \quad 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \\
\text{dB} & \quad -60 \quad -20 \quad 0 \quad 20 \quad 40 \quad 60 
\end{align*}$
A Delta-Sigma Beamformer

- No Interpolation
- Single-Bit Bus
Results
Cross-Section-Views of Peak 3

RF

Repeat

Insert-Zero

Symmetric-Hold
Generic Receiver

A/D -> beam former -> filtering (pre-detection) -> envelope detection -> filtering (post-detection) -> adaptive controls

display -> scan conversion

mapping and other processing
Scan Conversion

• Acquired data may not be on the display grid.
Scan Conversion

sinθ

acquired

converted
Scan Conversion

\[ p(m, n) = c_{m,n,i,j} a(i, j) + c_{m,n,i+1,j} a(i + 1, j) + c_{m,n,i,j+1} a(i, j + 1) + c_{m,n,i+1,j+1} a(i + 1, j + 1) \]
Moiré Pattern
Scan Conversion

original data buffer → interpolation → display buffer → display

addresses and coefficients generation
Temporal Resolution (Frame Rate)

- Frame rate = 1/Frame time.
- Frame time = number of lines * line time.
  - Line time = \((2 \times \text{maximum depth})/\text{sound velocity}\).
- Sound velocity is around 1540 m/s.
- High frame rate is required for real-time imaging.
Temporal Resolution

- The actual acoustic frame rate may be higher or lower. But should be high enough to have minimal flickering.
- Essence of real-time imaging: direct interaction.
Temporal Resolution

• For an actual frame rate lower than 30 Hz, interpolation is used.
• For an actual frame rate higher than 30 Hz, information can be displayed during playback.
• Even at 30 Hz, it is still possibly undersampling.
Temporal Resolution

- B-mode vs. Doppler.
- Acoustic power: peak vs. average.
- Increasing frame rate:
  - Smaller depth and width.
  - Less flow samples.
  - Wider beam width.
  - Parallel beam formation.
Parallel Beamformation

- Simultaneously receive multiple beams.
- Correlation between beams, spatial ambiguity.
- Require duplicate hardware (higher cost) or time sharing (reduced processing time and axial resolution).
Parallel Beamformation

• Simultaneously transmit multiple beams.
• Interference between beams, spatial ambiguity.

\[
\begin{align*}
n & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad t1/r1 \quad t2/r2 \\
\end{align*}
\]
Term Report